INTRODUCTION:

An Electric circuit is an interconnection of various elements in which there is at least one closed path in which current can flow. An Electric circuit is used as a component for any engineering system.

The performance of any electrical device or machine is always studied by drawing its electrical equivalent circuit. By simulating an electric circuit, any type of system can be studied for e.g., mechanical, hydraulic thermal, nuclear, traffic flow, weather prediction etc.

All control systems are studied by representing them in the form of electric circuits. The analysis, of any system can be learnt by mastering the techniques of circuit theory.

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Elements of an Electric circuit:

An Electric circuit consists of following types of elements.

Active elements:

Active elements are the elements of a circuit which possess energy of their own and can impart it to other element of the circuit.

Active elements are of two types

a) Voltage source b) Current source

A Voltage source has a specified voltage across its terminals, independent of current flowing through it.

A current source has a specified current through it independent of the voltage appearing across it.

Passive Elements:

The passive elements of an electric circuit do not possess energy of their own. They receive energy from the sources. The passive elements are the resistance, the inductance and the capacitance. When electrical energy is supplied to a circuit element, it will respond in one and more of the following ways.

If the energy is consumed, then the circuit element is a pure resistor.

If the energy is stored in a magnetic field, the element is a pure inductor.

And if the energy is stored in an electric field, the element is a pure capacitor.

Linear and Non-Linear Elements.

Linear elements show the linear characteristics of voltage & current. That is its voltage-current characteristics are at all-times a straight-line through the origin.

For example, the current passing through a resistor is proportional to the voltage applied through its and the relation is expressed as $V \propto I$ or $V = IR$. A linear element or network is one which satisfies the principle of superposition, i.e., the principle of homogeneity and additivity.

Resistors, inductors and capacitors are the examples of the linear elements and their properties do not change with a change in the applied voltage and the circuit current.

Non linear element's V-I characteristics do not follow the linear pattern i.e. the current passing through it does not change linearly with the linear change in the voltage across it. Examples are the semiconductor devices such as diode, transistor.

Bilateral and Unilateral Elements:

An element is said to be bilateral, when the same relation exists between voltage and current for the current flowing in both directions.

Ex: Voltage source, Current source, resistance, inductance & capacitance.

The circuits containing them are called bilateral circuits.

An element is said to be unilateral, when the same relation does not exist between voltage and current when current flowing in both directions. The circuits containing them are called unilateral circuits.

Ex: Vacuum diodes, Silicon Diodes, Selenium Rectifiers etc.

Lumped and Distributed Elements

Lumped elements are those elements which are very small in size & in which simultaneous actions takes place. Typical lumped elements are capacitors, resistors, inductors.

Distributed elements are those which are not electrically separable for analytical purposes.

For example a transmission line has distributed parameters along its length and may extend for hundreds of miles.

The circuits containing them are called unilateral circuits.

<u>Types of Sources:</u>

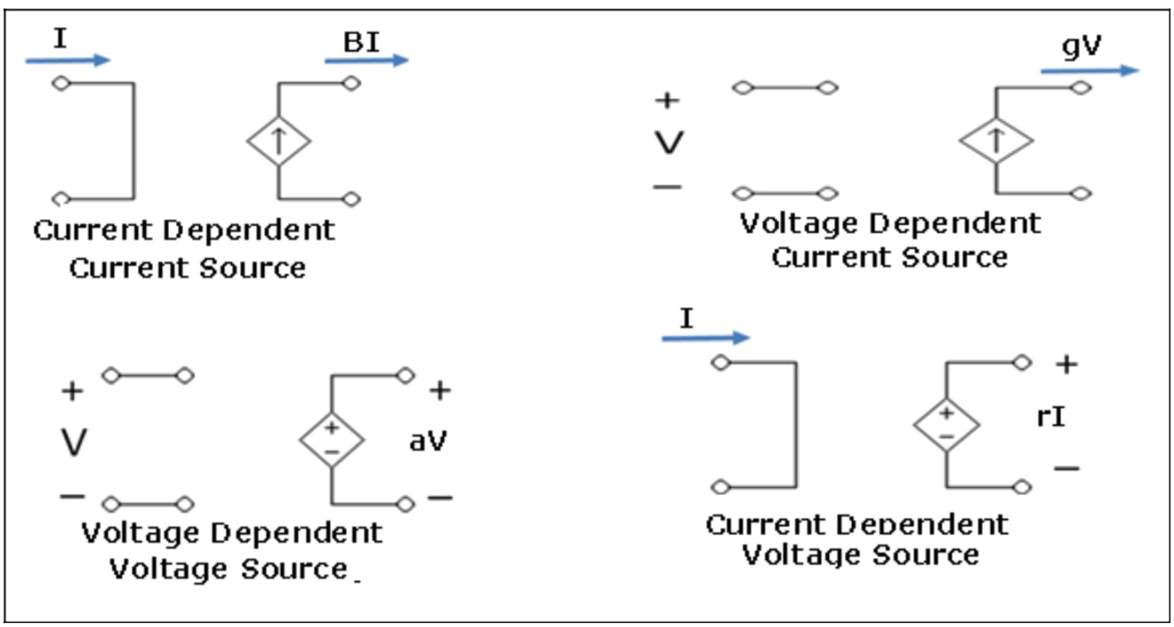
Independent & Dependent sources:

If the voltage of the voltage source is completely independent source of current and the current of the current source is completely independent of the voltage, then the sources are called as independent sources.

The special kind of sources in which the source voltage or current depends on some other quantity in the circuit which may be either a voltage or a current anywhere in the circuit are called Dependent sources or Controlled sources.

There are four possible dependent sources:

a. Voltage dependent Voltage source
b. Current dependent Current source
c. Voltage dependent Current source
d. Current dependent Current source

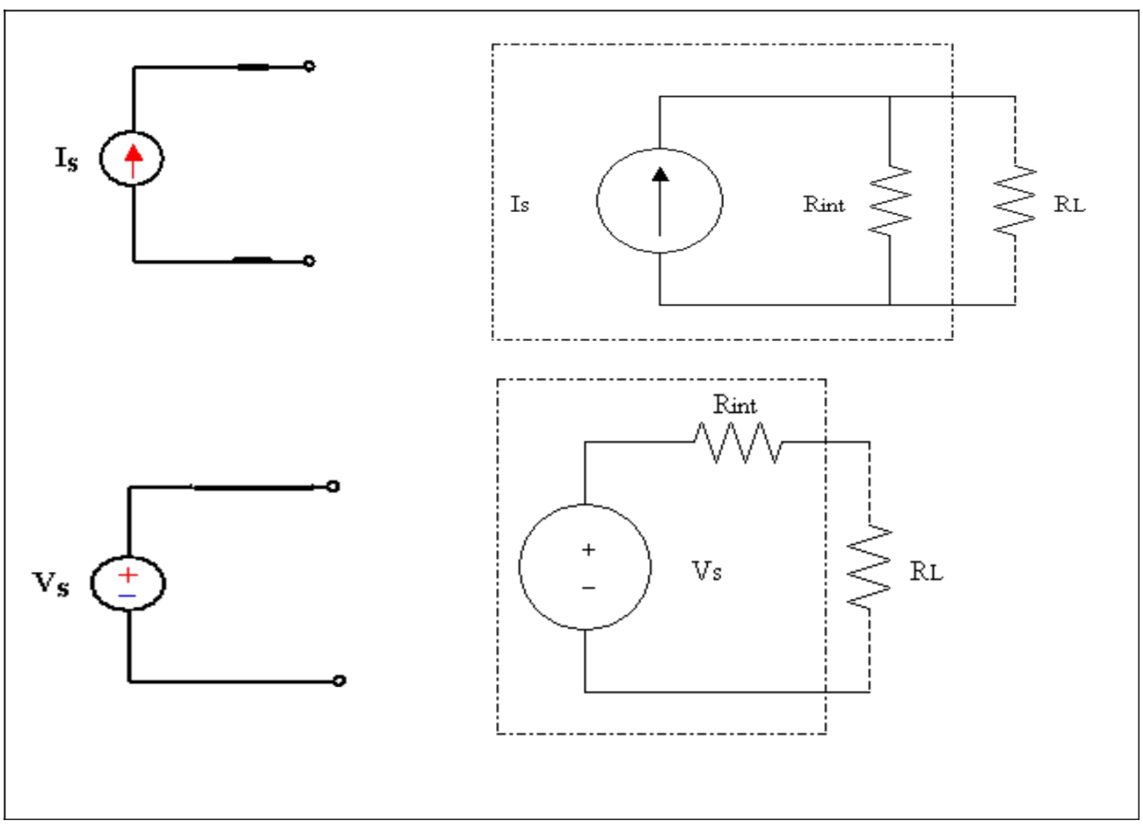


The constants of proportionalities are written as B, g, a, r in which B & a has no units, r has units of ohm & g units of mhos.

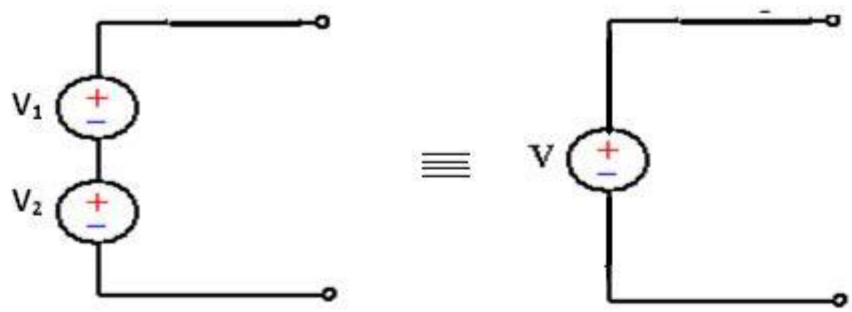
Independent sources actually exist as physical entities such as battery, a dc generator & an alternator. But dependent sources are used to represent electrical properties of electronic devices such as OPAMPS & Transistors.

Ideal & Practical sources:

1. An ideal voltage source is one which delivers energy to the load at a constant terminal voltage, irrespective of the current drawn by the load.
2. An ideal current source is one, which delivers energy with a constant current to the load, irrespective of the terminal voltage across the load.
3. A Practical voltage source always possesses a very small value of internal resistance r. The internal resistance of a voltage source is always connected in series with it & for a current source; it is always connected in parallel with it. As the value of the internal resistance of a practical voltage source is very small, its terminal voltage is assumed to be almost constant within a certain limit of current flowing through the load.
4. A practical current source is also assumed to deliver a constant current, irrespective of the terminal voltage across the load connected to it.



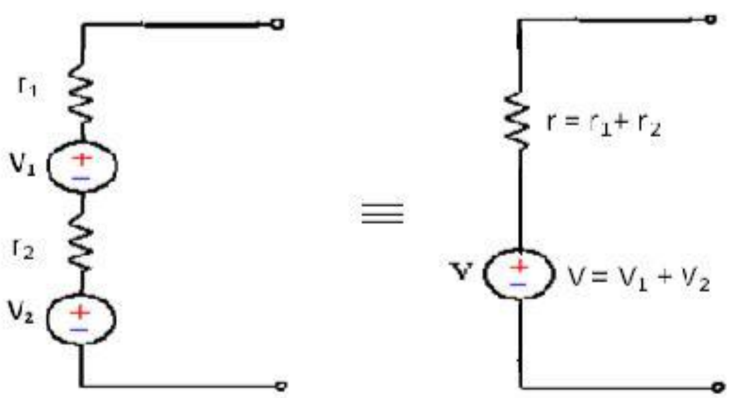
Ideal voltage source connected in series:



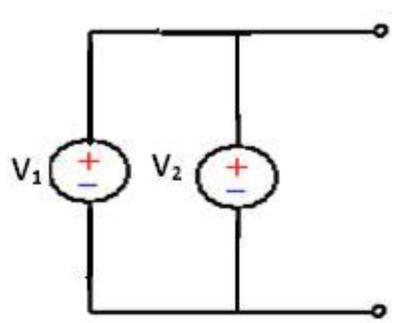
The equivalent single ideal voltage some is given by V= V1 + V2

Any number of ideal voltage sources connected in series can be represented by a single ideal voltage some taking in to account the polarities connected together in to consideration.

Practical voltage source connected in series:



Ideal voltage source connected in parallel:



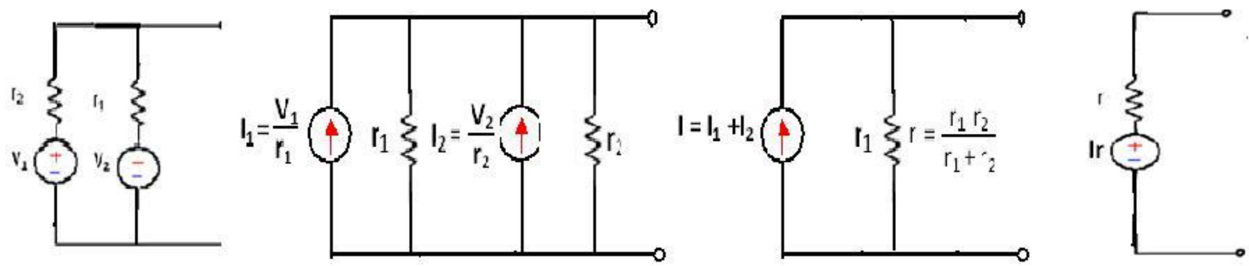
When two ideal voltage sources of emf's V1 & V2 are connected in parallel, what voltage appears across its terminals is ambiguous.

Hence such connections should not be made.

However if V1 = V2= V, then the equivalent voltage some is represented by V.

In that case also, such a connection is unnecessary as only one voltage source serves the purpose.

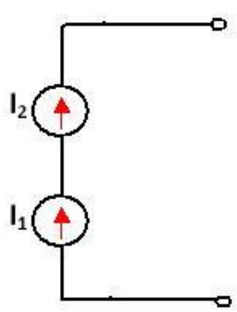
Practical voltage sources connected in parallel:



Equivalent Circuit

Single Equivalent Voltage Source

Ideal current sources connected in series:

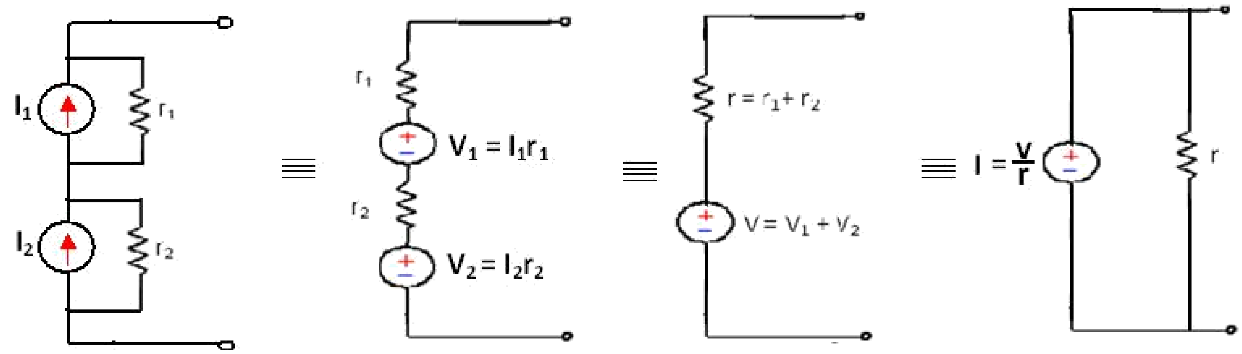


When ideal current sources are connected in series, what current flows through the line is ambiguous. Hence such a connection is not permissible.

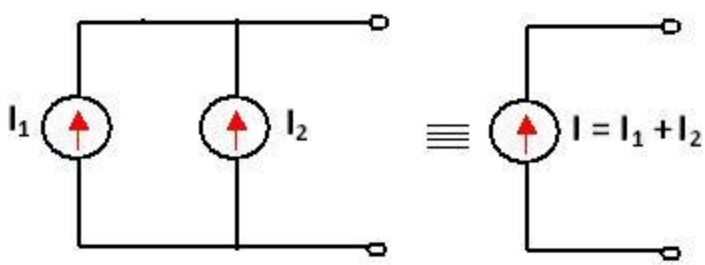
However, it I1 = I2 = I, then the current in the line is I.

But, such a connection is not necessary as only one current source serves the purpose.

Practical current sources connected in series:

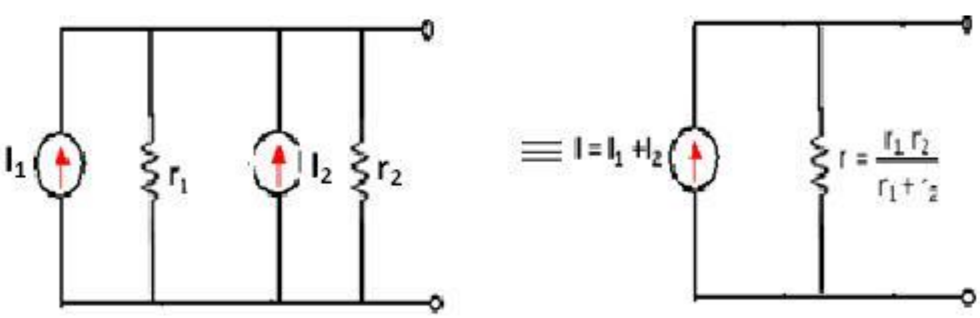


Ideal current sources connected in parallel



Two ideal current sources in parallel can be replaced by a single equivalent ideal current source.

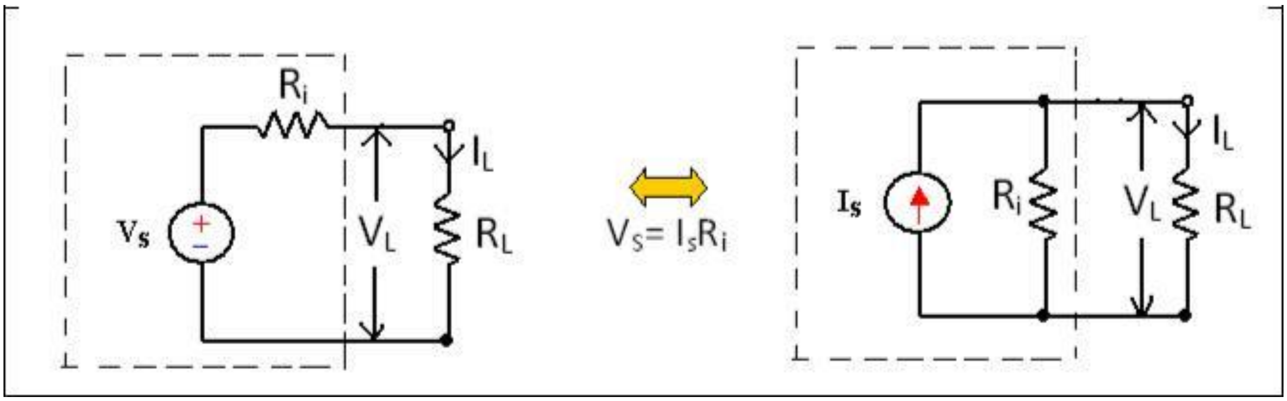
Practical current sources connected in parallel



Source transformation:

A current source or a voltage source drives current through its load resistance and the magnitude of the current depends on the value of the load resistance.

Consider a practical voltage source and a practical current source connected to the same load resistance RL as shown in the figure



R1's in figure represents the internal resistance of the voltage source VS and current source IS.

Two sources are said to be identical, when they produce identical terminal voltage VL and load current IL.

The circuit in figure represents a practical voltage source & a practical current source respectively, with load connected to both the sources.

The terminal voltage VL and load current IL across their terminals are same.

Hence the practical voltage source & practical current source shown in the dotted box of figure are equal.

The two equivalent sources should also provide the same open circuit voltage & short circuit current.

From fig (a)

IL = $\frac{Vs}{R+R_L}$

From fig (b)

IL = I $\frac{r}{R+R_L}$

$\therefore \frac{Vs}{R+R_L}$ = I

VS = IR or I = $\frac{Vs}{R}$

Hence a voltage source Vs in series with its internal resistance R can be converted into a current source $I = \frac{Vs}{R}$, with its internal resistance R connected in parallel with it. Similarly a current source I in parallel with its internal resistance R can be converted into a voltage source V = IR in series with its internal resistance R.

R-L-C Parameters:

1. **Resistance:**

Resistance is that property of a circuit element which opposes the flow of electric current and in doing so converts electrical energy into heat energy.

It is the proportionality factor in ohm's law relating voltage and current.

Ohm's law states that the voltage drop across a conductor of given length and area of cross section is directly proportional to the current flowing through it.

$$R \propto i$$

$$V=Ri$$

$$i=\frac{v}{R} = GV$$

Where the reciprocal of resistance is called conductance G. The unit of resistance is ohm and the unit of conductance is mho or Siemens.

When current flows through any resistive material, heat is generated by the collision of electrons with other atomic particles. The power absorbed by the resistor is converted to heat and is given by the expression

P= vi= i2R where i is the resistor in amps, and v is the voltage across the resistor in volts.

Energy lost in a resistance in time t is given by

$$W = \int_0^t p\,dt = pt = i^2Rt = \frac{v2}{R}t$$

2. **Inductance:**

Inductance is the property of a material by virtue of which it opposes any change of magnitude and direction of electric current passing through conductor. A wire of certain length, when twisted into a coil becomes a basic conductor. A change in the magnitude of the current changes the electromagnetic field.

Increase in current expands the field & decrease in current reduces it. A change in current produces change in the electromagnetic field. This induces a voltage across the coil according to Faradays laws of Electromagnetic Induction.

Induced Voltage $V = L\frac{di}{dt}$

V = Voltage across inductor in volts

I = Current through inductor in amps

$di = \frac{1}{L} v\, dt$

Integrating both sides,

$$\int_0^t di = \frac{1}{L}\int_0^t v dt$$

Power absorbed by the inductor $P = VI = Li\frac{di}{dt}$

Energy stored by the inductor

$$W = \int_0^t P\, dt = \int_0^t Li\, \frac{di}{dt}\, dt = \frac{Li^2}{2}$$

$$W = \frac{Li^2}{2}$$

Conclusions:

1) $V = L\frac{di}{dt}$

   The induced voltage across an inductor is zero if the current through it is constant. That means an inductor acts as short circuit to dc.

2) For minute change in current within zero time (dt = 0) gives an infinite voltage across the inductor which is physically not at all feasible. In an inductor, the current cannot change abruptly. An inductor behaves as open circuit just after switching across dc voltage.

3) The inductor can store finite amount of energy, even if the voltage across the inductor is zero.

4) A pure inductor never dissipates energy, it only stores it. Hence it is also called as a non–dissipative passive element. However, physical inductor dissipates power due to internal resistance.

Ex: The current in a 2H inductor raises at a rate of 2A/s .Find the voltage across the inductor the energy stored in the magnetic field at after 2sec.

Sol:

$$V = L\frac{di}{dt}$$

$$= 2X2 = 4V$$

$W = \frac{1}{2} Li^2 = \frac{1}{2} \times 2 \times (4)^2 = 16\ J$

3. Capacitance:

1) A capacitor consists of two metallic surfaces or conducting surfaces separated by a dielectric medium.
2) It is a circuit element which is capable of storing electrical energy in its electric field.
3) Capacitance is its capacity to store electrical energy.
4) Capacitance is the proportionality constant relating the charge on the conducting plates to the potential.

Charge on the capacitor q ⍰ V

$$q = CV$$

Where `C` is the capacitance in farads, if q is charge in coulombs and V is the potential difference across the capacitor in volts.
The current flowing in the circuit is rate of flow of charge

$i = \frac{dq}{dt} = C\frac{dv}{dt}$ $\therefore i = C\frac{dv}{dt}$

The capacitance of a capacitor depends on the dielectric medium & the physical dimensions. For a parallel plate capacitor, the capacitance

$$\mathbf{C = \frac{€A}{D} = €_0\, €_r \frac{A}{D}}$$

A is the surface area of plates D is the separation between plates

€ is the absolute permeability of medium€0 is the absolute permeability of free space €r is the relative permeability of medium

$i = \frac{dq}{dt} = C\frac{dv}{dt}$

$\frac{dv}{dt} = \frac{i}{c}$

$\mathbf{V = \frac{1}{c}\int i\,dt}$

The power absorbed by the capacitor $P = vi = vc\frac{dv}{dt}$

Energy stored in the capacitor $W = \int_0^t P\,dt = \int_0^t VC\frac{dv}{dt}dt$

$= C\int_0^t v\,dv = \frac{1}{2}cv^2$ **Joules**

This energy is stored in the electric field set up by the voltage across capacitor.

Conclusions:

1. The current in a capacitor is zero, if the voltage across it is constant, that means the capacitor acts as an open circuit to dc
2. A small change in voltage across a capacitance within zero time gives an infinite current through the capacitor, which is physically impossible.
   - ∴ In a fixed capacitor, the voltage cannot change abruptly
   - ∴ A capacitor behaves as short circuit just after switching across dc voltage.
3. The capacitor can store a finite amount of energy, even if the current through it is zero.
4. A pure capacitor never dissipates energy but only stores it hence it is called non-dissipative element.

Kirchhoff`s Laws:

Kirchhoff's laws are more comprehensive than Ohm's law and are used for solving electrical networks which may not be readily solved by the latter.

Kirchhoff`s laws, two in number, are particularly useful in determining the equivalent resistance of a complicated network of conductors and for calculating the currents flowing in the various conductors.

1. **Kirchhoff`s Current Law (KCL)**

In any electrical network, the algebraic sum of the currents meeting at a point (or junction) is Zero.

That is the total current ***entering*** a junction is equal to the total current ***leaving*** that junction.

Consider the case of a network shown in Fig (a*).*

$I_1+(-I_2)+(I_3)+(+I_4)+(-I_5) = 0$

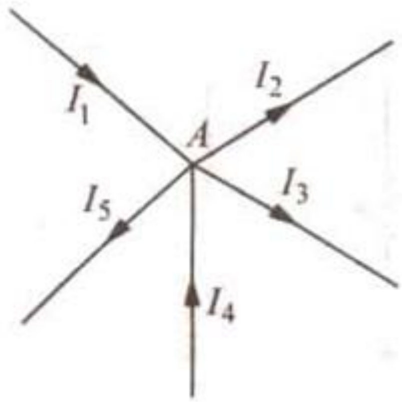
$I_1+I_4-I_2-I_3-I_5 = 0$

Or

$I_1+I_4 = I_2+I_3+I_5$

Or

Incoming currents =Outgoing currents



2. Kirchhoff's Mesh Law or Voltage Law (KVL)

In any electrical network, the algebraic sum of the products of currents and resistances in each of the conductors in any closed path (or mesh) in a network plus the algebraic sum of the e.m.f.'s. in that path is zero.

That is, $\sum IR + \sum e.m.f = 0$ round a mesh

It should be noted that algebraic sum is the sum which takes into account the polarities of the voltage drops.

That is, if we start from a particular junction and go round the mesh till we come back to the starting point, then we must be at the same potential with which we started.

Hence, it means that all the sources of emf met on the way must necessarily be equal to the voltage drops in the resistances, every voltage being given its proper sign, plus or minus.

Determination of Voltage Sign

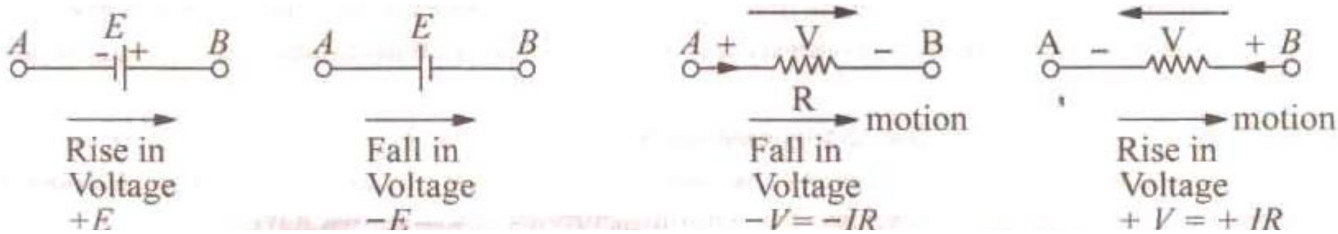
In applying Kirchhoff's laws to specific problems, particular attention should be paid to the algebraic signs of voltage drops and e.m.fs.

(a) **Sign of Battery E.M.F.**

A *rise* in voltage should be given a + ve sign and *a fall* in voltage a -ve sign. That is, if we go from the -ve terminal of a battery to its +ve terminal there is a *rise* in potential, hence this voltage should be given a + ve sign.

And on the other hand, we go from +ve terminal to -ve terminal, then there is *a fall* in potential, hence this voltage should be preceded by a -ve sign.

The sign of the battery e.m.f is independent of the direction

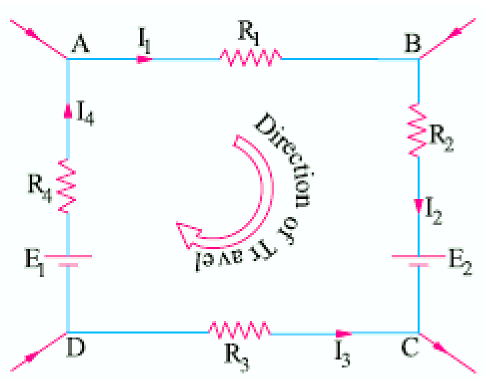


of the current through that branch.

***(b)* Sign of *IR* Drop**

Now, take the case of a resistor (Fig. 2.4). If we go through a resistor in the *same* direction as the
current, then there is a fall in potential because current flows from a higher to a lower potential..
Hence, this voltage fall should be taken -ve. However, if we go in a direction opposite to that of the
current, then there is a *rise* in voltage. Hence, this voltage rise should be given a positive sign.

Consider the closed path *ABCDA* in Fig .



As we travel around the mesh in the clockwise direction, different voltage drops will have the following signs :

I_1R_1 is - ve (fall in potential)
I_2R_2 is - ve (fall in potential)
I_3R_3 is + ve (rise in potential)
I_4R_4 is - ve (fall in potential)
E_2 is - ve (fall in potential)
E_1 is + ve (rise in potential)

Using Kirchhoff's voltage law, we get

$$-I_1R_1 - I_2R_2 - I_3R_3 - I_4 R_4 - E_2 + E_1 = 0$$

Or $$I_1R_1 + I_2R_2 - I_3R_3 + I_4R_4 = E_1 - E_2$$

Assumed Direction of Current:

In applying Kirchhoff's laws to electrical networks, the direction of current flow may be assumed either clockwise or anticlockwise. If the assumed direction of current is not the actual direction, then on solving the question, the current will be found to have a minus sign.

If the answer is positive, then assumed direction is the same as actual direction. However, the important point is that once a particular direction has been assumed, the same should be used throughout the solution of the question.

Kirchhoff's laws are applicable both to d.c. and a.c. voltages and currents. However, in the case of alternating currents and voltages, any e.m.f. of self-inductance or that existing across a capacitor should be also taken into account.

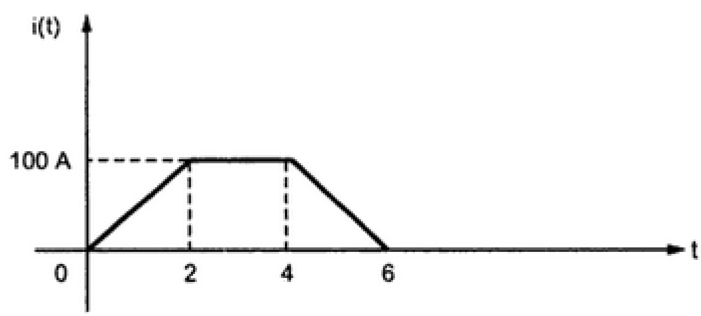
VOLTAGE-CURRENT RELATIONSHIPS FOR PASSIVE ELEMENTS

The Three Passive Elements are Resistance, Inductance and Capacitance. The behavior of these three elements along with the respective voltage-current relationship is given in the table.

Element	Basic Relation	Voltage across, If Current Known	Current through, If Voltage Known
R	R=$\frac{V}{I}$	$V_R(t)=R\, i_R(t)$	$i_R(t)=\frac{V_R(t)}{R}$
L	L=$\frac{N\phi}{I}$	$V_L(t)=L\frac{di(t)}{dt}$	$i_L(t)=\frac{1}{L}\int_{-\infty}^{t} V(t)dt$
C	$C=\frac{Q}{V}$	$V_C(t)=\frac{1}{C}\int_{-\infty}^{t} i_C(t)dt$	$i_C(t)=C\frac{dv(t)}{dt}$

Table

1. A current waveform flowing through an inductor of 1mH is shown in the figure. Obtain and sketch the waveform of voltage across the inductor.



Solution:

From the given waveform,

For 0<t<2, i(t) is a straight line of slope=(100/2)=50

Therefore, i(t)=50t and $\frac{di(t)}{dt}$=50

For 2<t<4, i(t)=100 and $\frac{di(t)}{dt}$=0

For 4<t<6, i(t) is a straight line of slope=-(100/2)=-50

Therefore, i(t)=-50t and $\frac{di(t)}{dt}$=-50

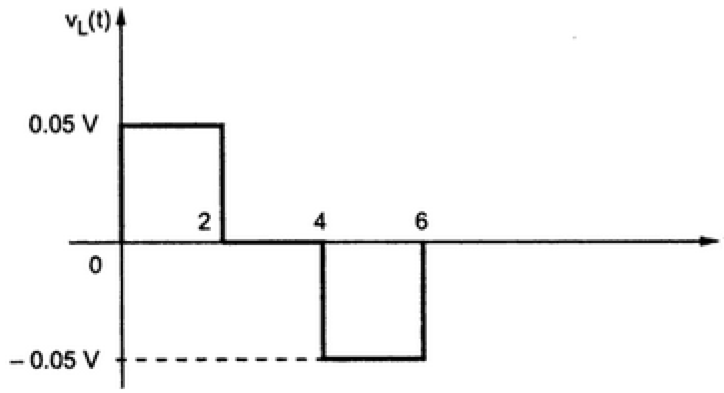
Now, $V_L(t) = L\frac{di(t)}{dt}$

$=1*10^{-3}*50=0.05V$ 0<t<2

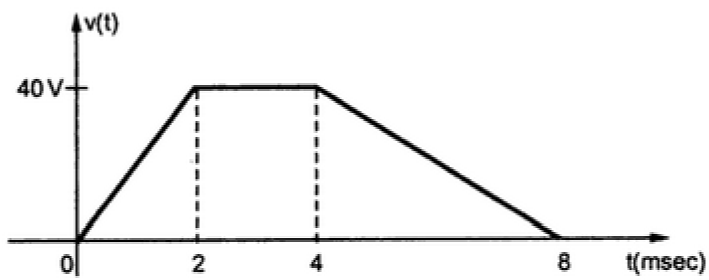
$=1*10^{-3}*0=0V$ 2<t<4

$=1*10^{-3}*(-50) = -0.05V$ 4<t<6

The voltage waveform is shown in following figure.



2. A 0.5uF capacitor has voltage waveform v(t) as shown in following figure, plot i(t) as function of t ?



Solution:

From the given waveform,

For 0<t<2, v(t) is a ramp of slope =(40/2)=20

Therefore $v(t)=20t$

Therefore $i(t)=C\frac{dv(t)}{dt}=0.5*10^{-6}*20=1*10^{-5}A=10uA$

For 2<t<4, v(t) is constant

Therefore $v(t)=40V$

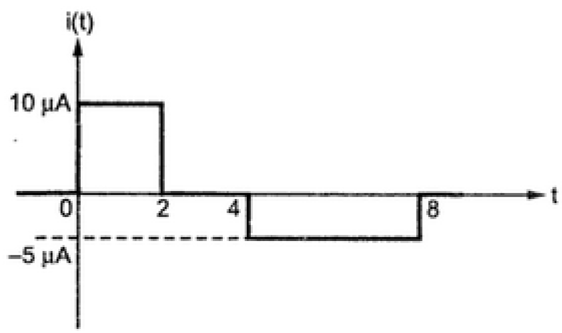
Therefore $i(t)=C\frac{dv(t)}{dt}$=0.5*10^{-6}*0=0A

For 4<t<8, v(t) is a ramp with slope = $\frac{0-40}{8-4} = -10$

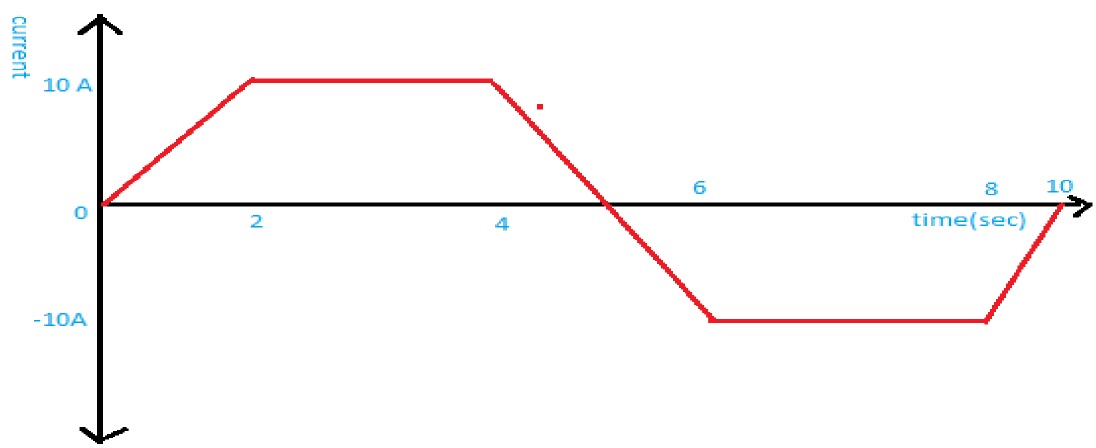
Therefore v(t)=-10t+80 (According to straight line equation i.e. y=mx+c)

Therefore $i(t)=C\frac{dv(t)}{dt}$=0.5*10^{-6}*(-10)=-5uA

The current waveform is shown in following figure



3. A Pure Inductance Of 3mh Carries A Current Of The Waveform Shown In Fig. Sketch The Waveform Of V (t) And P(t).Determine The Average Value Of Power



Fig

Solution:

i (t)=5t for 0<t<2

i (t)=10 for 2<t<4

i (t)=-10t+50 for 4<t<6

i (t)=-10 for 6<t<8

$i(t) = 5t-50$ for $8<t<10$

For $0<t<2$, $V_L(t) = L\frac{di(t)}{dt}$

$= 3*10^{-3}\frac{d(5t)}{dt}$

$= 15*10^{-3}$v

For $2<t<4$, $V_L(t) = L\frac{di(t)}{dt}$

$= 3*10^{-3}\frac{d(10)}{dt}$

$= 0$v

For $4<t<6$, $V_L(t) = L\frac{di(t)}{dt}$

$= 3*10^{-3}\frac{d(-10t+50)}{dt}$

$= -30*10^{-3}$v

For $6<t<8$, $V_L(t) = L\frac{di(t)}{dt}$

$= 3*10^{-3}\frac{d(-10)}{dt}$

$= 0$v

For $8<t<10$, $V_L(t) = L\frac{di(t)}{dt}$

$= 3*10^{-3}\frac{d(5t-50)}{dt}$

$= 15*10^{-3}$v

The sketch of v(t) is shown in fig.

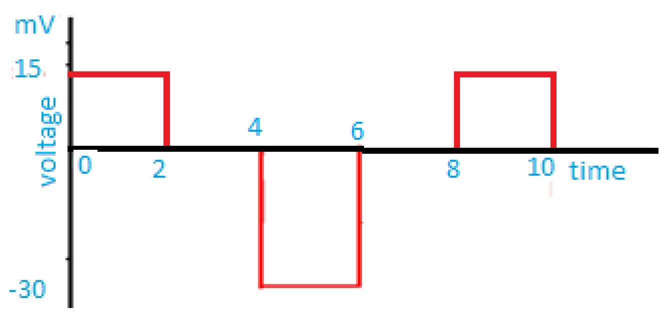


Fig.

For 0<t<2, P(t)=v(t)i(t)

=75*10-3t W

For 2<t<4, P(t)=v(t)i(t)

=0W

For 4<t<6, P(t)=v(t)i(t)

$=-30*(-10t+50)*10^{-3}=-0.3W$ (at t=4)

=0W (at t=5)

=0.3W (at t=6)

For 6<t<8, P(t)=v(t)i(t)

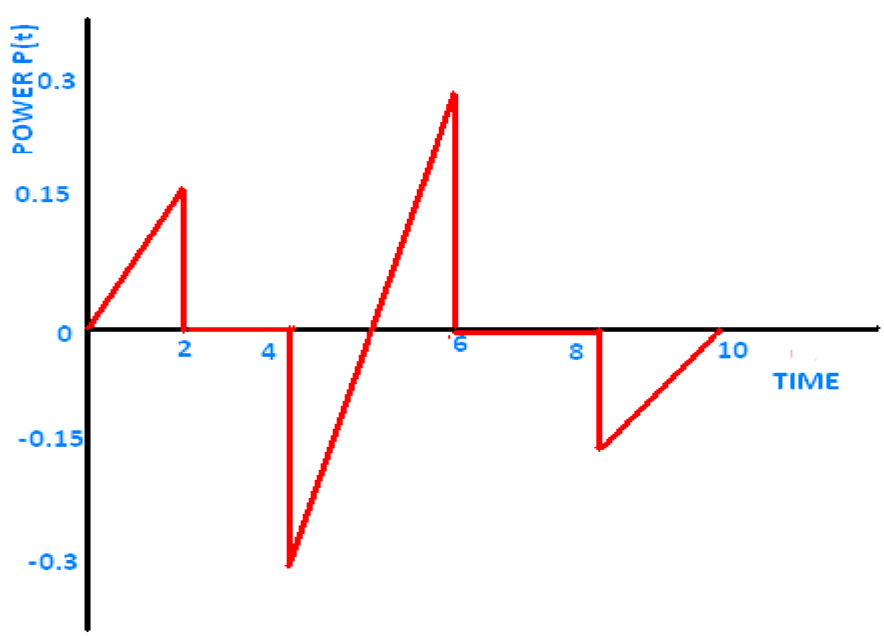
=0W

For 8<t<10, P(t)=v(t)i(t)

$=15*(5t-50)*10^{-3}=-0.15W$ (at t=8)

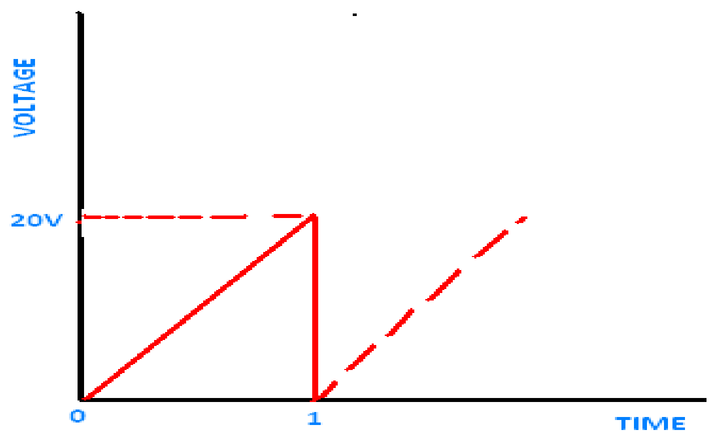
=-0.075W (at t=9)

=0W (at t=10)



4. Draw the waveforms for current, power for the following elements if a voltage input shown in figure is applied to these elements.

i. R=1 OHM
ii. L=1H
iii. C=1F



SOLUTION:

From the figure, v (t) is a straight line with slope = $\frac{20-0}{1-0}$=20, For 0<t<1

Therefore v (t) =20t

i. **R=1 OHM**

The voltage and current relation of a resistor is given by, v (t) =R i(t)

i(t) =20t/1=20t

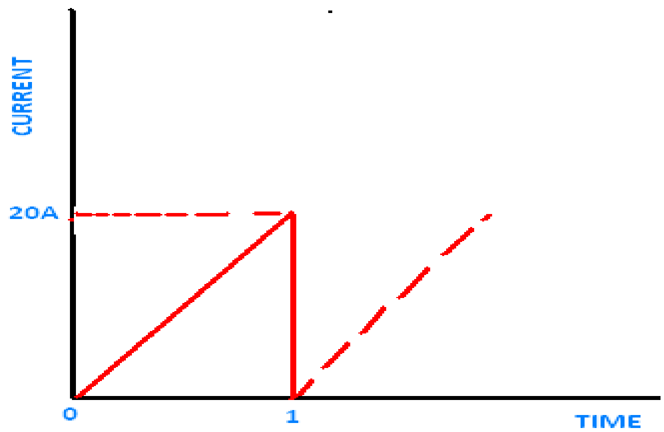
Hence,

When t=0, i(t)=0A

When t=0.5, i(t)=10A

When t=1, i(t)=20A

Therefore the current waveform for the above values of t and i(t) is shown in figure below



Power, p(t)=v(t)i(t)

=20t*20t=400t^2 W

Hence,

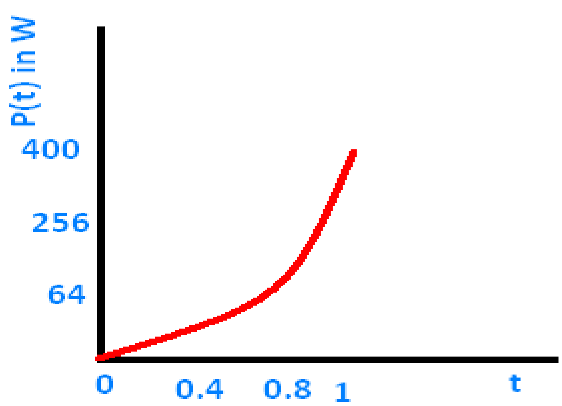
When t=0, p(t)=v(t)i(t)=0W

When t=0.4, p(t)=v(t)i(t)=64W

When t=0.8, p(t)=v(t)i(t)=256W

When t=, p(t)=v(t)i(t)=400W

Therefore the power wave form for the above values of t and p(t) is shown in below figure.



ii. **L=1 H**

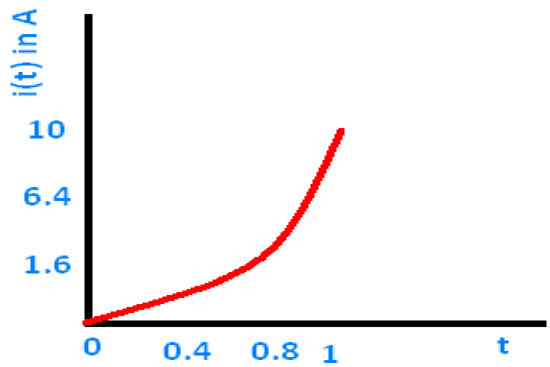
The voltage and current relation of a inductor is given by,

$$i(t)=\frac{1}{L}\int_{-\infty}^{t} V(t)dt$$

$$i(t)=\frac{1}{1}[\int_{-\infty}^{0} V(t)dt + \int_{0}^{t} V(t)dt]$$

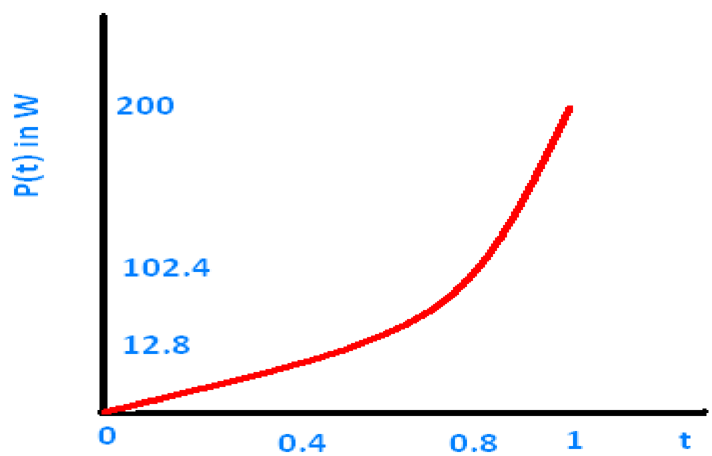
$$i(t)=0+\int_0^t V(t)dt=\int_0^t 20t dt=10t^2$$

Therefore the current waveform is shown in below figure.



Power, $p(t)=v(t)i(t)=20t*10t^2=200t^3$ W

Therefore the power waveform is shown in below figure



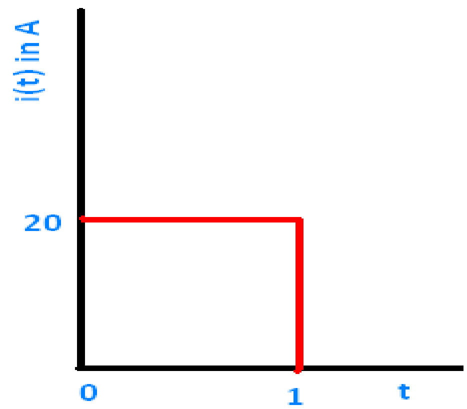
iii. **C=1 F**

The voltage and current relation of a inductor is given by

$$i(t)=C\frac{dV(t)}{dt}$$

$$i(t)=1*\frac{d(20t)}{dt}=20A$$

Therefore the current waveform is shown in below figure



Power, p(t)=v(t)i(t)=20t*20=400t W

Therefore the power waveform is shown in below figure.

NETWORK ANALYSIS

- **Introduction**
- **Network Reduction Techniques**
- **Resistive Networks, Inductive Networks and Capacitive Networks**
- **Series, Parallel, and Series Parallel Connections**
- **Star to Delta and Delta to Star Transformations**
- **Mesh Analysis and Super Mesh for DC excitation**
- **Nodal Analysis and Super Node for DC excitation**
- **Network Topology Definitions: Graph, Tree, and Basic Tie-set, Basic Cut-set Matrices for planar Networks**

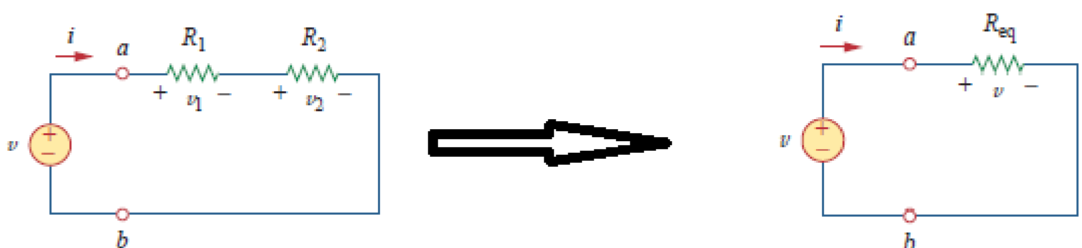
Introduction:

A network is a collection of interconnected electrical components. In general, the electrical networks are made to exchange the energy between different elements .These electrical networks can be constructed either by using Resistors or Inductors or Capacitors or combination of these elements. Network analysis is the process of finding the voltage response or the current response for any element in the network by using the available techniques.

Network Reduction Techniques:

Series Connection of Resistors:

Two or more resistors in a circuit are said to be in series when the current flowing through all the resistors is the same.



fig(a) fig(b)

Consider the circuit in fig(a) ,where two resistors R_1 and R_2 are in series, since the same current i flows in both of them. Applying Ohm's law to each of the resistors, we obtain

$v_1 = iR_1$,.......(1) $v_2 = iR_2$........(2)

If we apply KVL to the loop fig(b) , we have

$-v + v_1 + v_2 = 0$..........................(3)

$v = v1 + v2 = i(R1 + R2)$

$i = v/(R1 + R2)$ ....*(4)*

$v = i\,Req$................(5)

Implying that the two resistors can be replaced by an equivalent resistor; that is,

$$Req = R1 + R2$$

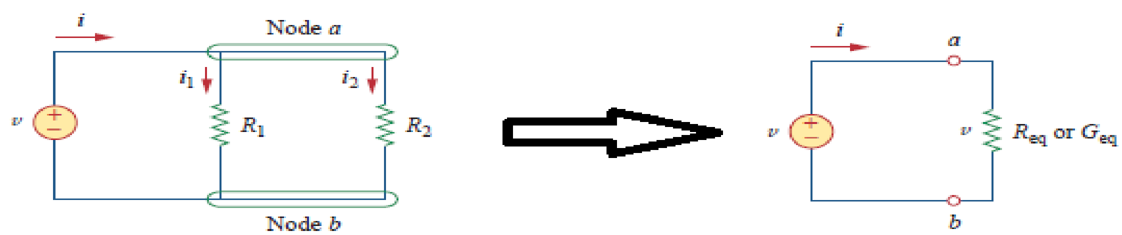
Note: The equivalent resistance of any number of resistors connected in series is the sum of the individual resistances.

If "n"resistors are in series, Req=R1+R2+......Rn

If "n"resistors of same value are in series, Req=nR

Parallel Connection of Resistors:

Two or more resistors in a circuit are said to be in Parallel when all the resistors are connected to the same nodes and the same voltage is appearing across all these elements.



fig(a) **fig(b)**

Consider the circuit in fig(a), where two resistors are connected in parallel and therefore have the same voltage across them. From Ohm’s law,

$v = i_1R_1 = i_2R_2$..................(1)

$i1 = v/R1$, $i2 = v/R2$..................(2)

Applying KCL at node a gives the total current i as

$i = i1 + i2$................(3)

Substituting Eq. (2) into Eq. (3), we get

$i = v/R_1 + v/R_2 = v(1/R_1 + 1/R_2) = v/Req$

$1/Req = 1/R_1 + 1/R_2$

$$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$$

Note: The equivalent resistance of two parallel resistors is equal to the product of their resistances divided by their sum.

If "n"resistors are in parallel, $1/Req = 1/R1 + 1/R2 + ...1/Rn$

If "n"resistors of same value are in parallel, $Req = R/n$

It is often more convenient to use conductance rather than resistance when dealing with resistors in parallel. The equivalent conductance for N resistors in parallel is

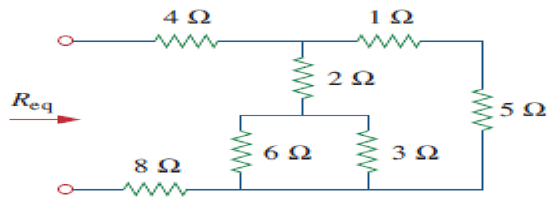
$$G_{eq} = G_1 + G_2 + G_3 + \cdots + G_N$$

where $G_{eq} = 1/R_{eq}$, $G_1 = 1/R_1$, $G_2 = 1/R_2$, $G_3 = 1/R_3$, $\ldots$, $G_N = 1/R_N$.

Note: The equivalent conductance of resistors connected in parallel is the sum of their individual conductances.

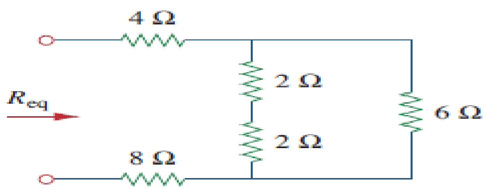
Example Problems:

1) Find the Req for the circuit shown in below figure.

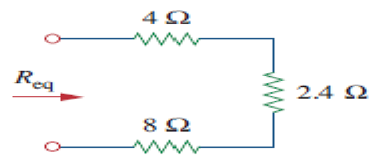


fig(a)

Solution:



fig(b)



fig(c)

To get Req we combine resistors in series and in parallel. The 6 ohms and 3 ohms resistors are in parallel, so their equivalent resistance is

$$6\,\Omega \parallel 3\Omega = \frac{6 \times 3}{6 + 3} = 2\,\Omega$$

Also, the 1 ohm and 5ohms resistors are in series; hence their equivalent resistance is

$$1\,\Omega + 5\,\Omega = 6\,\Omega$$

Thus the circuit in Fig.(b) is reduced to that in Fig. (c). In Fig. (b), we notice that the two 2 ohms resistors are in series, so the equivalent resistance is

$$2\,\Omega + 2\,\Omega = 4\,\Omega$$

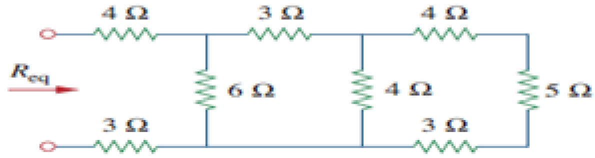
This 4 ohms resistor is now in parallel with the 6 ohms resistor in Fig.(b); their equivalent resistance is

$$4\,\Omega \parallel 6\,\Omega = \frac{4 \times 6}{4 + 6} = 2.4\,\Omega$$

The circuit in Fig.(b) is now replaced with that in Fig.(c). In Fig.(c), the three resistors are in series. Hence, the equivalent resistance for the circuit is

$$R_{eq} = 4\,\Omega + 2.4\,\Omega + 8\,\Omega = 14.4\,\Omega$$

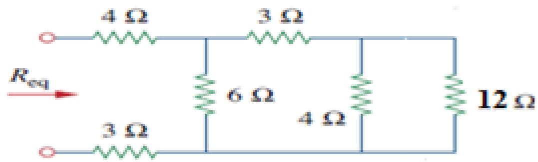
2) Find the Req for the circuit shown in below figure.



fig(a)

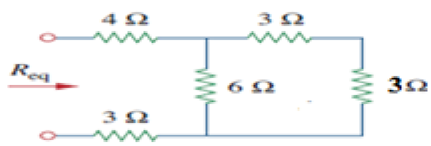
Solution:

In the given network 4 ohms, 5 ohms and 3 ohms comes in series then equivalent resistance is 4+5 + 3 = 12 ohms



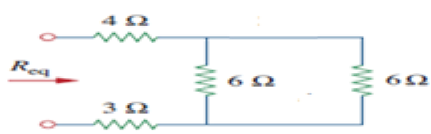
fig(b)

From fig(b), 4 ohms and 12 ohms are in parallel, equivalent is 3 ohms



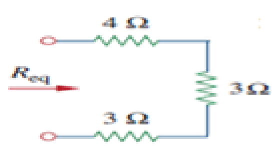
fig(c)

From fig(c), 3 ohms and 3 ohms are in series, equivalent resistance is 6 ohms



fig(d)

From fig(d), 6 ohms and 6 ohms are in parallel, equivalent resistance is 3 ohms

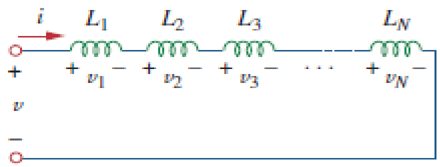


fig(e)

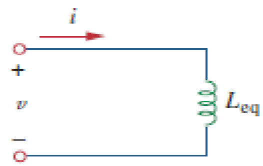
From fig(e), 4 ohms, 3 ohms and 3 ohms are in series .Hence Req = 4+ 3+ 3 =10 ohms

Series Connection of Inductors:

Two or more inductors in a circuit are said to be in series when the current flowing through all the inductors is the same.



fig(a)



fig(b)

Consider a series connection of *N* inductors, as shown in Fig(a),with the equivalent circuit shown in Fig(b). The inductors have the same current through them. Applying KVL to the loop,

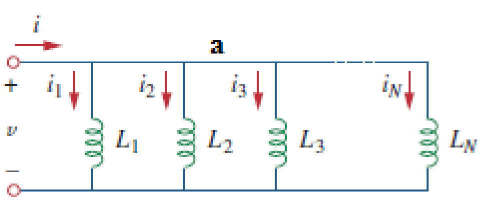
V = V1 + V2 + ……+Vn = L1 . di/dt + L2 . di/dt + …… Ln . di/dt
= di/dt (L1 +L2+ … +Ln)
= di/dt(L eq)

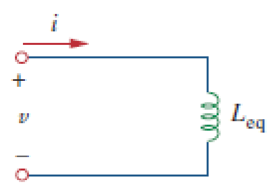
where Leq is the equivalent Inductance of all the Inductances L1 ,L2 ,….and Ln in series. Hence Leq of a series circuit consisting of n Inductances L1 ,L2, … Ln connected in series is given by

Leq = L1 +L2+ … +Ln

Note: The equivalent Inductance Leq of a circuit with n Inductances L1, L2, ….Ln connected in series is equal to the sum of the individual Inductances L1 ,L2, … Ln

Parallel Connection of Inductors:





fig(b)

fig(a)

Two or more inductors in a circuit are said to be in Parallel when all the circuit inductors are connected to the same nodes and the same voltage is appearing across all these inductors

Consider the circuit in fig(a) and using the Kirchhoff's Current Law (KCL) at the node 'a' the governing equation can be written as:

i = i1 + i2 + ……+in..........(1)

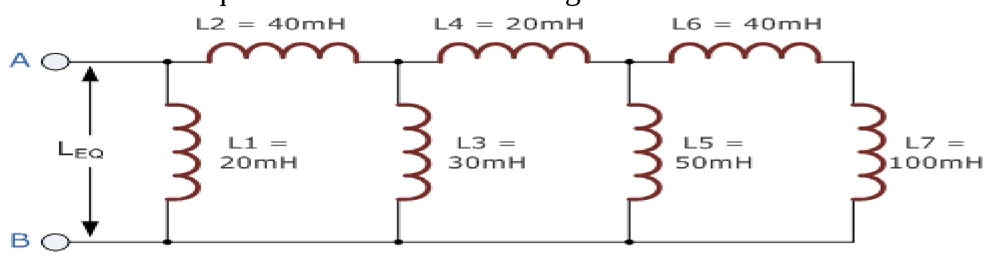
In terms of the applied voltage V , the individual Inductances L1 ,L2, … Ln the above equation can be written as:

i = (1/L1 $\int$ vdt +(1/L2) $\int$ vdt + … +(1/Ln) $\int$ vdt = [(1/L1) +(1/L2) + … +(1/Ln)] $\int$ vdt.....(2)

Similarly for the circuit in fig(b) we can write the governing equation as:
i = (1/Leq) $\int$ v....(3)
where Leq is the equivalent Inductance of all the Inductances L1 , L2 ,….and Ln in parallel.
Since current is the same in the above two equations we find that
1/ Leq = (1/ L1 + 1/ L2 + ….. 1/ Ln)
Hence Leq of a parallel circuit consisting of n Inductances L1 , L2 ,….and Ln connected in parallel is given by :

1/ Leq = (1/ L1 + 1/ L2 + ….. 1/ Ln)

Note: The reciprocal of the equivalent inductance is the sum of the reciprocals of the inductances.
Problem: Determine the equivalent inductance in the given network.



Solution:
Calculating the first inductor branch L_A, where an Inductor L_5 in parallel with inductors L_6 and L_7.

$$LA = \frac{L5 \times (L6 + L7)}{L5 + L6 + L7} = \frac{50mH \times (40mH + 100mH)}{50mH + 40mH + 100mH} = 36.8mH$$

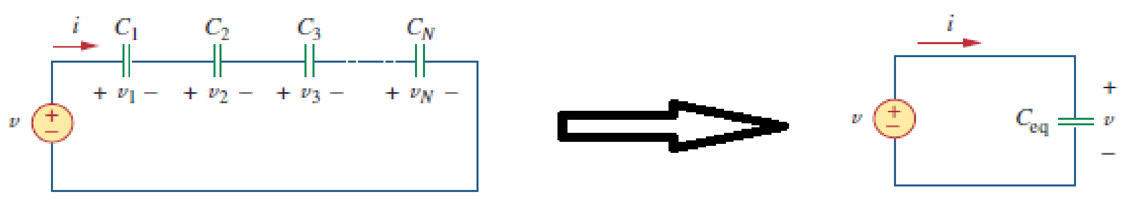
Calculating the second inductor branch L_B, where Inductor L_3 in parallel with inductors L_4 and L_A.

$$LB = \frac{L3 \times (L4 + LA)}{L3 + L + LA} = \frac{30mH \times (20mH + 36.8mH)}{30mH + 20mH + 36.8mH} = 19.6mH$$

Calculate the equivalent circuit inductance LEQ, where Inductor L_1 in parallel with inductors $L_2$and L_B.

$$LEQ, = \frac{L1\times(L2+LB)}{L1+L2+LB} = \frac{20mH\times(40mH+19.6mH)}{20mH+40mH+19.6mH} = 15mH$$

Series Connection of Capacitors:



fig(a) **fig(b)**

Two or more capacitors in a circuit are said to be in series when the current flowing through all the capacitors is the same.

Fig (a) above shows n capacitances C1 ,C2, … Cn connected in series along with a Voltage source V and V1 , V2 …… Vn are the voltage drops across the capacitances C1 , C2 ,….and Cn. Fig (b) shows the same circuit with it's equivalent Capacitance Ceq.

Applying KVL to the circuit in figure (a) we can write: v = v1 + v2 + v3 + ….+ vn............(1)
V = (1/C1) ∫ idt +(1/C2) ∫ idt + … +(1/Cn) ∫ idt = [(1/C1) +(1/C2) + … +(1/Cn)] ∫ idt.........(2)

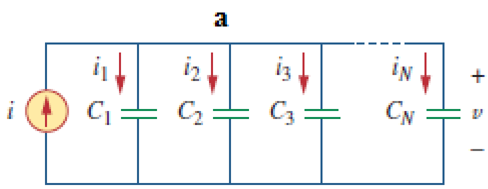
Similarly for the circuit in fig.(b) we can write the governing equation as:
v = (1/Ceq) ∫idt................(3)
From the above two equations i.e eq(2) and eq (3) for 'v' we get
1/Ceq = (1/C1) +(1/C2) + … +(1/Cn)
Hence Ceq of a circuit consisting of n capacitances C1 , CL2 ,….and Cn connected in series is given by :

1/Ceq = (1/C1) +(1/C2) + … +(1/Cn)

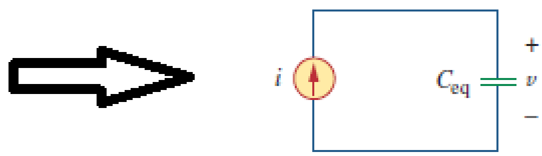
Note: The reciprocal of the equivalent capacitance is the sum of the reciprocals of the inductances**.**

Parallel Connection of Capacitors:

Two or more capacitors in a circuit are said to be in Parallel when all the capacitors are connected to the same nodes and the same voltage is appearing across all these capacitors



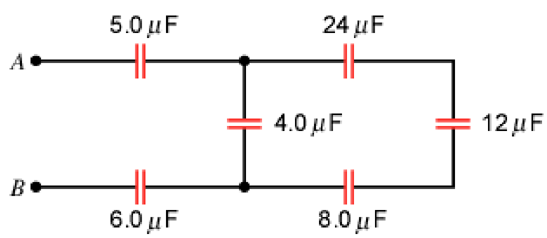
fig(a)



fig(2)

Applying KCL to the circuit in figure (a) we can write: i = i1 + i2 + i3 + ….+ in......(1)
i = C1 (dv/dt) +C2 (dv/dt) + … + Cn dv/dt) = (C1 + C2 + … + Cn) (dv/dt).............(2)
Similarly for the circuit in figure (b) we can write the governing equation as:
i = Ceq (dv/dt).............(3)
From the eq(2) and eq(3),we get
Ceq=C1+C2+....+Cn
Note: The equivalent capacitance of a circuit consisting of n capacitances C1 , CL2 ,….and Cn connected in parallel is the sum of the individual capacitances.

Problem: Determine the equivalent capacitance between AB in the given circuit.



Solution: In the given circuit 24uF,12uF and 8uF are in series. The equivalent capacitance of this three capacitors is

$$\frac{1}{C_s} = \frac{1}{24\ \mu\text{F}} + \frac{1}{12\ \mu\text{F}} + \frac{1}{8.0\ \mu\text{F}} \qquad \text{or} \qquad C_s = 4.0\ \mu\text{F}$$

This 4.0μF capacitance is in parallel with the 4.0μF capacitance. Then the equivalent capacitance is

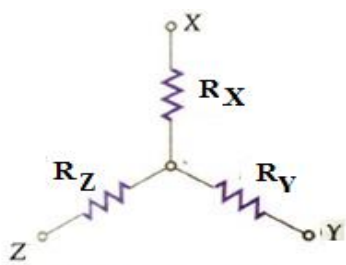
$$C_p = 4.0\ \mu\text{F} + 4.0\ \mu\text{F} = 8.0\ \mu\text{F}$$

This 8.0μF,5.0uF and 6.0uF are in series .Hence , the overall equivalent capacitance is

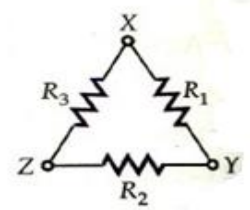
$$\frac{1}{C_s} = \frac{1}{5.0\,\mu\text{F}} + \frac{1}{8.0\,\mu\text{F}} + \frac{1}{6.0\,\mu\text{F}} \qquad \text{or} \qquad C_s = \boxed{2.0\,\mu\text{F}}$$

Star to Delta and Delta to Star Transformations:

Like in series and parallel connections, electrical components may be connected in Star or Delta configurations as shown in the figure below (with Resistances). Many a times circuits have to be transformed from Star to equivalent Delta and Delta to equivalent Star configurations such that the net terminal Resistances (Impedances)across the terminals are the same. We will show this transformation methodology and the resulting configurations for both Delta to Star and Star to Delta one by one.



(a) Star connection



(b) Delta connection

Delta to Star Transformation:

The circuit configurations are identical provided the net resistances across the terminal pairs XY, YZ and ZX in both connections are the same. In Star Connection they are:

$\mathbf{R_{X\text{-}Y} = R_X + R_Y}$ ....(1)

$\mathbf{R_{Y\text{-}Z} = R_Y + R_Z}$ ....(2)

$\mathbf{R_{Z\text{-}X} = R_Z + R_X}$ ....(3)

Similarly in Delta connection they are:

$$\mathbf{R_{X\text{-}Y} = R1//(R2+R3) = \frac{R1(R2+R3)}{R1+R2+R3}} \quad \text{........................(4)}$$

$$\mathbf{R_{Y\text{-}Z} = R2//(R1+R3) = \frac{R2(R1+R3)}{R1+R2+R3}} \quad \text{..........................(5)}$$

$$\mathbf{R_{Z\text{-}X} = R3//(R1+R2) = \frac{R3(R1+R2)}{R1+R2+R3}} \quad \text{............................(6)}$$

By equating the respective equations, we get

$$\mathbf{R_X+R_Y = \frac{R1(R2+R3)}{R1+R2+R3}} \quad \textbf{...........(7)}$$

$$\mathbf{R_Y+R_Z = \frac{R2(R1+R3)}{R1+R2+R3}} \quad \textbf{.............(8)}$$

$$\mathbf{R_Z+R_X = \frac{R3(R1+R2)}{R1+R2+R3}} \quad \textbf{.............(9)}$$

By subtracting equation 8 from equation 7 given above, we get

$$RX - RZ = \frac{R1R2 + R1R3}{R1 + R2 + R3} - \frac{R2R1 + R2R3}{R1 + R2 + R3} \ldots . \ldots \ldots . (10)$$

Then adding this equation to equation 9 above i.e. (R_Z+R_X) we get:

$$2RX = \frac{R1R2 + R1R3 - R2R1 - R2R3 + R3R1 + R3R2}{R1 + R2 + R3}$$

$$= \frac{2R1R3}{R1 + R2 + R3}$$

$$RX = \frac{R1R3}{R1 + R2 + R3}$$

And in a similar way we can get:

$$RY = \frac{R1R2}{R1 + R2 + R3}$$

$$RZ = \frac{R2R3}{R1 + R2 + R3}$$

Where RX, RY and RZ are the equivalent resistances in the Star connection corresponding to the Delta connection with resistances R1, R2 and R3.

Star to Delta Transformation:

Now we have to get the equivalent values of R1, R2 and R3 in Delta connection in terms of the three resistances R_X,R_Y and R_Z in Star connection.

Let us use the equations we got earlier i.e. R_X,R_Y and R_Z in terms of R_1,R_2 and R_3 and get the sum of the three product pairs i.e. $R_XR_Y + R_YR_Z + R_ZR_X$ as :

$$RXRY + RYRZ + RZRX = \frac{R1^2R2R3 + R2^2R1R3 + R3^2R1R2}{(R1 + R2 + R3)^2}$$

Now let us divide this equation by R_X to get:

$$RY + RZ + \frac{RYRZ}{RX} = \frac{R1R2R3(R1 + R2 + R3)}{RX(R1 + R2 + R3)^2}$$
$$= \frac{R1R2R3}{RX(R1 + R2 + R3)}$$

Now substituting the value of R_X = (R1+R2+R3) / R1.R3 from the earlier equations into the above equation we get:

$$RY + RZ + \frac{RYRZ}{RX} = \frac{R1R2R3}{(R1 + R2 + R3)} \times \frac{(R1 + R2 + R3)}{R1R3} = R2$$

Then similarly dividing the same equation by RY and RZ we get the other two relations as:

$$RX + RZ + \frac{RXRZ}{RY} = \mathrm{R3}$$

$$RY + RX + \frac{RXRY}{RZ} = R1$$

Thus we get the three equivalent resistances R1, R2 and R3 in Delta connection in terms of the three resistances RX, RY and RZ in Star connection as :

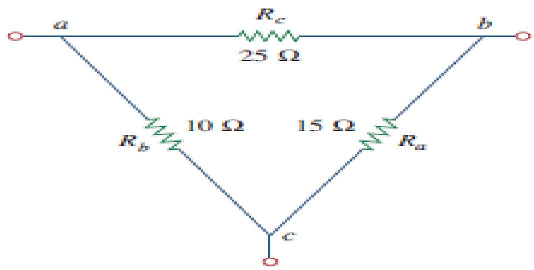
$$RY + RX + \frac{RXRY}{RZ} = R1$$

$$RY + RZ + \frac{RYRZ}{RX} = R2$$

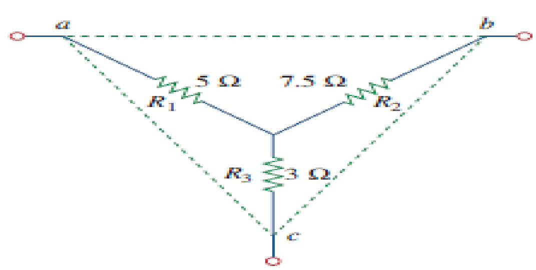
$$RX + RZ + \frac{RXRZ}{RY} = \mathrm{R3}$$

Example problems:

1) Convert the Delta network in a) Fig.(a) to an equivalent star network



(a)



(b)

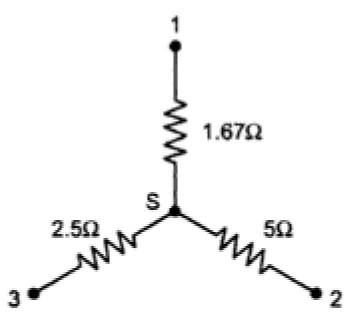
Solution:

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{10 \times 25}{15 + 10 + 25} = \frac{250}{50} = 5\ \Omega$$

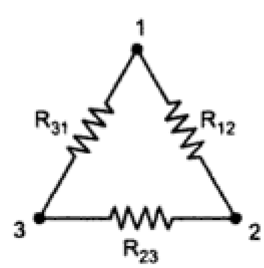
$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} = \frac{25 \times 15}{50} = 7.5\ \Omega$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{15 \times 10}{50} = 3\ \Omega$$

2) Convert the star network in fig(a) to delta network



fig(a)



fig(b)

Solution: The equivalent delta for the given star is shown in fig(b), where

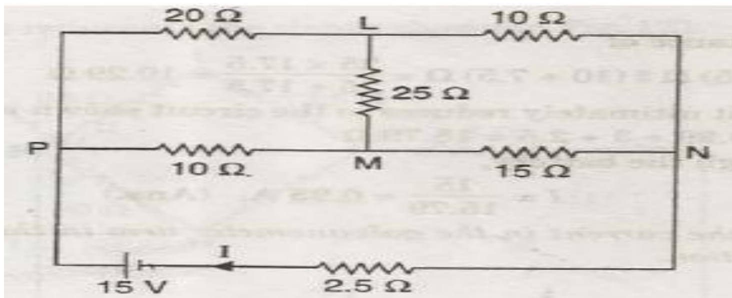
$$R_{12} = 1.67 + 5 + \frac{1.67 \times 5}{2.5} = 1.67 + 5 + 3.33 = \mathbf{10\ \Omega}$$

$$R_{23} = 5 + 2.5 + \frac{5 \times 2.5}{1.67} = 5 + 2.5 + 7.5 = \mathbf{15\ \Omega}$$

$$R_{31} = 2.5 + 1.67 + \frac{2.5 \times 1.67}{5} = 2.5 + 1.67 + 0.833$$

$$= \mathbf{5\ \Omega}$$

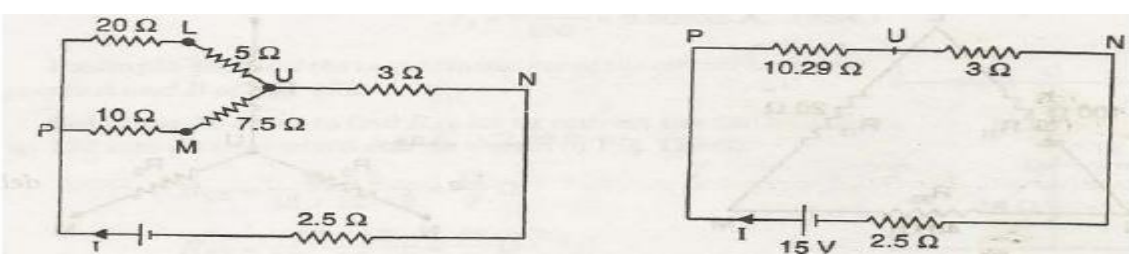
3) Determine the total current I in the given circuit.



Solution: Delta connected resistors 25 ohms, 10 ohms and 15 ohms are converted in to star as shown in given figure**.**

$R_1 = R_{12}\ R_{31} / R_{12} + R_{23} + R_{31}$ = 10 x 25 / 10 + 15 + 25 = 5 ohms
$R_2 = R_{23}\ R_{12} / R_{12} + R_{23} + R_{31}$ = 15 x 10 / 10 + 15 + 25 = 3 ohms
$R_3 = R_{31}\ R_{23} / R_{12} + R_{23} + R_{31}$ = 25 x 15 / 10 + 15 + 25 = 7.5 ohms

The given circuit thus reduces to the circuit shown in below fig.

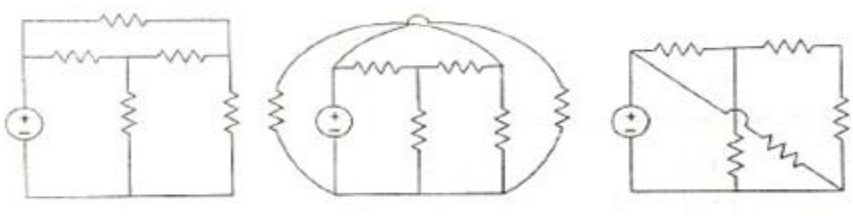


The equivalent resistance of
(20 + 5) ohms || (10 + 7.5) ohms = 25 x 17.5 / 25 + 17.5 = 10.29 ohms
Total resistance = 10.29 + 3 + 2.5 = 15.79 ohms
Hence the total current through the battery,
I = 15 / 15.79 = 0.95 A

Introduction to Mesh Analysis and Nodal Analysis:
Mesh Analysis and Nodal Analysis are two important techniques used in network analysis to find out different branch currents and Node voltages. The suitability of each analysis depends mainly on the number of voltage/current sources in the given network. If the voltage sources are more Mesh analysis is suitable and if current sources are more Nodal analysis is more suitable.

Mesh Analysis:
Mesh analysis provides general procedure for analyzing circuits using mesh currents as the circuit variables. Mesh Analysis is applicable only for planar networks. It is preferably useful for the circuits that have many loops .This analysis is done by using KVL and Ohm's law.

Planar circuit: A planar circuit is one that can be drawn in a plane with no branches crossing one another. In the figure below (a) is a planar circuit.
Non-Planar circuit: A planar circuit is one that cannot be drawn in a plane without the branches crossing one another. In the figure below (b) is a n.on-planar circuit and (c) is a planar circuit but appears like a non-planar circuit

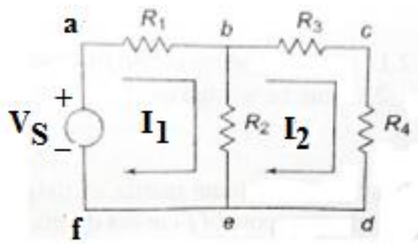

(a) (b) (c)

:

Loop: It is a closed path along the circuit elements.

Mesh: Mesh is a loop which does not contains any loop within it.

Mesh analysis with example:
Determination of mesh currents:



Solution:

Step (1): Identify the no. of meshes in the given circuit.

There are two meshes..Mesh (1)......**abef** and

Mesh (2).....**bcde**

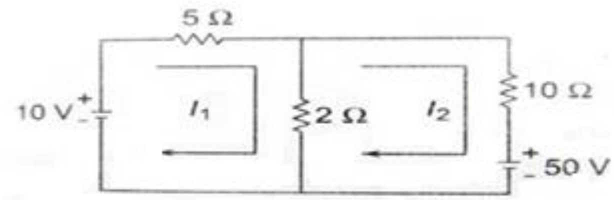
Step (2): Apply the KVL to the all meshes.

For mesh (1) by applying KVL… **Vs -I1 .R1 + (I1- I2). R2 =0...........(1)**

For mesh (2) by applying KVL**....** **I2 .(R3)+ I2 (R4)+ (I2- I1). R2=0.......(2**)

Step (3): solve the above equations for mesh currents.

Problem: Write down the mesh current equations for the circuit shown in the figure below and determine the currents I1 and I2.



Solution:

By applying KVL to the two meshes, we get

5 I1 + 2(I1- I2)=10

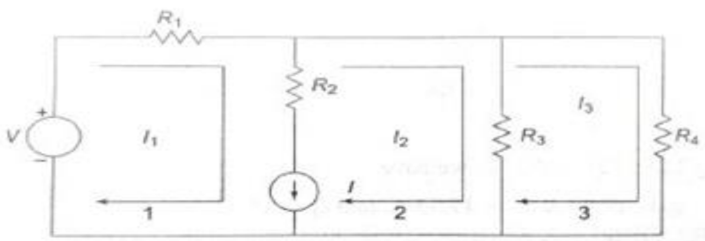
10 I2 + 2(I2- I1)=-50.

Solving the above equations gives.... I1 = 0.25 A and I2 = -4.25 A. The negative sign for the current I2 indicates that it flows in the opposite direction to that assumed in the loop two.

Super Mesh Analysis: If there is only current source between two meshes in the given network then it is difficult to apply the mesh analysis. Because the current source has to be converted into a voltage source in terms of the current source, write down the mesh equations and relate the mesh currents to the current source. But this is a difficult approach .This difficulty can be avoided by creating super mesh which encloses the two meshes that have common current source

Super Mesh: A super mesh is constituted by two adjacent meshes that have a common current source.

Let us illustrate this method with the following simple generalized circuit.



Solution:

Step (1): Identify the position of current source.

Here the current source is common to the two meshes 1 and 2. so, super mesh is nothing but the combination of meshes 1 and 2 .
Step (2): Apply KVL to super mesh and to other meshes
Applying KVL to this super mesh (combination of meshes 1 and 2) we get

$R1.I1 + R3 (I2 – I3) = V$..............(1)

Applying KVL to mesh 3, we get

$R3 (I3 – I2) + R4.I3 = 0$............(2)

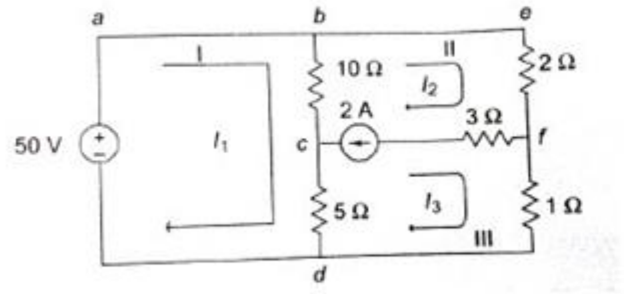
Step (3): Make the relation between mesh currents with current source to get third equation.

Third equation is nothing but he relation between I , I1 and I2 which is

$I1 - I2 = I$............(3)

Step(4): Solve the above equations to get the mesh currents.

Example(1): Determine the current in the 5 Ω resistor shown in the figure below.



Solution:

Step(1): Here the current source exists between mesh(2) and mesh(3).Hence, super mesh is the combination of mesh(2) and mesh(3) .Applying KVL to the super mesh (combination of mesh 2 and mesh 3 after removing the branch with the current source of 2 A and resistance of 3 Ω) we get :

10(I2– I1) + 2.I2 + I3 + 5(I3 – I1) = 0

-15.I1 +12 I2 + 6.I3 = 0...................(1)

Step (2): Applying KVL first to the normal mesh 1 we get :

10(I1 – I2) + 5(I1 – I3) = 50

15.I1 –10. I2 – 5.I3 = 50....................(2)

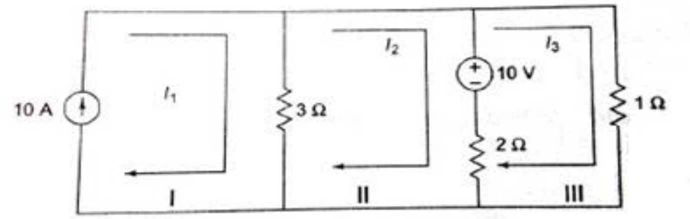
Step (3): We can get the third equation from the relation between the current source of 2 A , and currents I2 & I3 as :

I2 - I3 = 2 A...................(3)

Step (4): Solving the above three equations for I1, I2 and I3 we get I1 = 19.99 A I2 = 17.33 A and I3 = 15.33 A

The current in the 5 Ω resistance = I1 - I3 = 19.99 - 15.33 = 4.66 A

Example(2): Write down the mesh equations for the circuit shown in the figure below and find out the values of the currents I1, I2 and I3



Solution: In this circuit the current source is in the perimeter of the circuit and hence the first mesh is ignored. So, here no need to create the super mesh.

Applying KVL to mesh 1 we get :

3(I2 – I1) + 2(I2 – I3) = -10

-3.I1 +5.I2 – 2.I3 = -10..............(1)

Next applying KVL to mesh 2 we get :

I3 + 2(I3– I2) = 10

-2.I2 +3.I3 = -10...............(2)

And from the first mesh we observe that....... I1 = 10 A..............(3)

And solving these three equations we get : I1 = 10 A, I2 = 7.27 A, I3 = 8.18 A

Nodal analysis:

Nodal analysis provides another general procedure for analyzing circuits nodal voltages as the circuit variables. It is preferably useful for the circuits that have many no. of nodes. It is applicable for the both planar and non planar circuits. This analysis is done by using KCL and Ohm's law.

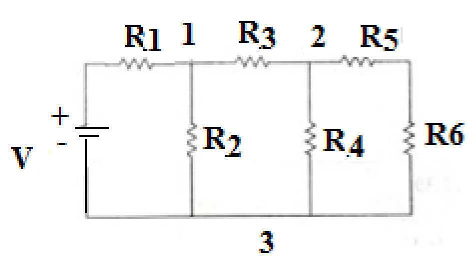
Node: It is a junction at which two or more branches are interconnected.

Simple Node: Node at which only two branches are interconnected.

Principal Node: Node at which more than two branches are interconnected.

Nodal analysis with example:

Determination of node voltages:

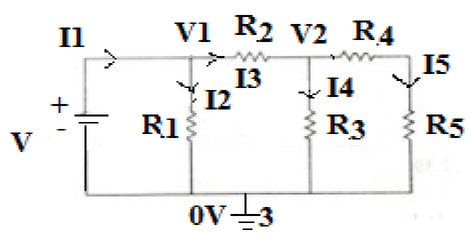


Procedure:

Step (1): Identify the no. nodes, simple nodes and principal nodes in the given circuit. Among all the nodes one node is taken as reference node. Generally bottom is taken as reference node. The potential at the reference node is 0v.

In the given circuit there are 3 principal nodes in which node (3) is the reference node.

Step (2): Assign node voltages to the all the principal nodes except reference node and assign branch currents to all branches.



Step (3): Apply KCL to those principal nodes for nodal equations and by using ohm's law express the node voltages in terms of branch current.

Applying KCL to node (1)----**I1=I2+I3**

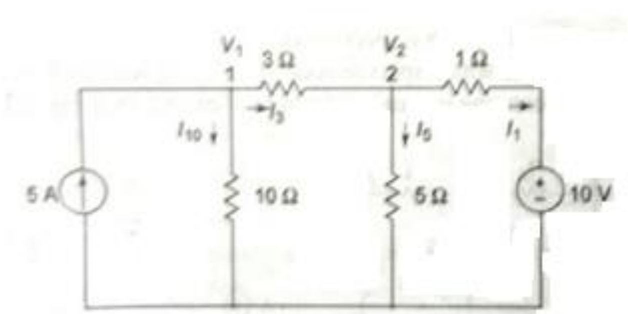
Using ohm's law, we get **(V-V1)/R1 =(V2-0)/R2 +(V1-V2)/R3..........(1)**

Applying KCL to node (2)----**I3=I4 +I5**

Using ohm's law, we get **(V1-V2)/R3 =(V4-0)/R4 +(V5-0)/R5 ....(2)**

Step(4): Solve the above nodal equations to get the node voltages.

Example: Write the node voltage equations and find out the currents in each branch of the circuit shown in the figure below.



Solution:

The node voltages and the directions of the branch currents are assigned as shown in given figure.
Applying KCL to node 1, we get: **5 = I10+ I3**

5= (V1-0)/10 +(V1-V2)/3

V1(13/30) -V2(1/3) = 5 ....(1)

Applying KCL to node 2, we get: **I3= I5 + I1**

(V1-V2)/3 = (V2 -0)/5 + (V2-10) /1

V1(1/3)-V2(23/15) = -10................(2)

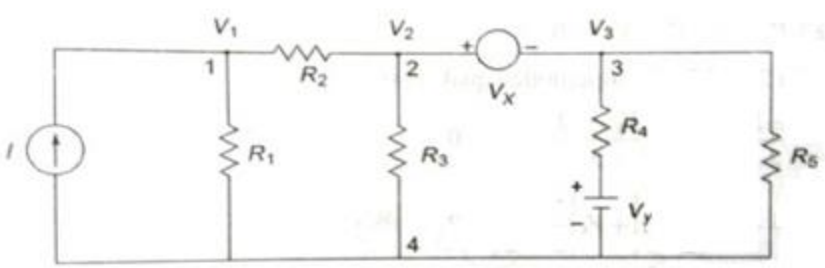
Solving the these two equations for V1 and V2 we get :

V1 = 19.85 V and V2 = 10.9 V and the currents are :
I10= V1/10 = 1.985A
I3 = (V1-V2)/3 = (19.85-10.9)/3 = 2.98A
I5 = V2/5 = 10.9/5 =2.18A
I1 = (V2-10) = (10.9-10)/1 = 0.9A

Super Node Analysis: If there is only voltage source between two nodes in the given network then it is difficult to apply the nodal analysis. Because the voltage source has to be converted into a current source in terms of the voltage source, write down the nodal equations and relate the node voltages to the voltage source. But this is a difficult approach .This difficulty can be avoided by creating super node which encloses the two nodes that have common voltage source.

Super Node: A super node is constituted by two adjacent nodes that have a common voltage source.

Example: Write the nodal equations by using super node analysis.



Procedure:

Step(1): Identify the position of voltage source.Here the voltage source is common to the two nodes 2 and 3.so, super node is nothing but the combination of nodes 2 and 3 .

Step (2): Apply KCL to super node and to other nodes.
Applying KCL to this super node (combination of meshes 2 and 3), we get

(V2-V1)/R2 + V2/R3 + (V3-Vy)/R4 + V3/R5 = 0...............(1)

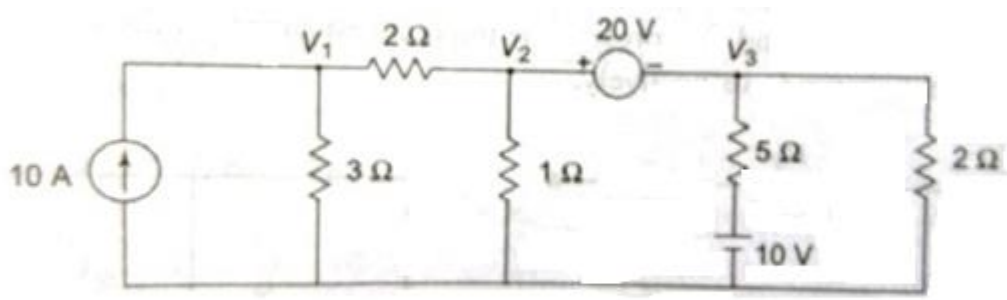
Applying KVL to node 1 ,we get

I = V1/R1 + (V1-V2)/R2.................(2)

Step (3): Make the relation between node voltages with voltage source to get third equation.

Third equation is nothing but the relation between VX , V2 and V3 which is
V2 - V3 = Vx..............(3)
Step (4): Solve the above nodal equations to get the node voltages.

Example: Determine the current in the 5 Ω resistor shown in the circuit below



Solution:

Applying KCL to node 1: 10 = V1/3 + (V1-V2)/2
V1 [1/3 + 1/2] - V2 /2 = 10

0.83 V1 - 0.5 V2 = 10.................(1)

Next applying KCL to the super node2&3 :
(V2-V1)/2 + V2/1 + (V3-10)/5 + V3/2 = 0
-V1/2 + V2(1/2 + 1) V3(1/5 + 1/2) = 2
0.5 V1 + 1.5V2 + 0.7 V3 = 2........................(2)
and the third and final equation is:
V2 - V3 = 20.....................(3)

Solving the above three equations we get V3 = -8.42 V

The current through the 5 Ω resistor I5 = [-8.42 -10] /5 = -3.68 A
The negative sign indicates that the current flows towards the node 3.